

## Inverted Re-Entrant Turnstile Waveguide Circulator Using Prism Resonator with Arbitrary Orientation

Joe Helszajn<sup>1</sup>, John Sharp<sup>2</sup>, Alicia Casanueva<sup>3</sup>, \*, Angel Mediavilla<sup>3</sup>, and Marco Caplin<sup>4</sup>

**Abstract**—The waveguide re-entrant or inverted re-entrant turnstile circulator relies for its operation on either a quarter wave long cylindrical or prism resonator mounted on either a circular or an equilateral platform with its open face separated from the top waveguide wall by a suitable gap. The adjustment of the prism geometry, the main endeavour of this paper, is characterized by two degrees of freedom. One degree of freedom is the orientation of the prism inside the junction with respect to a typical waveguide feed. The others are the aspect ratio of the gyromagnetic resonator and the choice of platform or piston. The agreement between some calculations based on a Finite Element (FE) engine and experiment is excellent. The work undertaken here indicates that the preferred geometry is that for which the platform has the crosshairs section of the resonator.

### 1. INTRODUCTION

The 3-port junction circulator is an essential component in the design of all microwave hardware. It relies for its operation on either, a cylindrical, or one of two possible orientations of a prism gyromagnetic resonator, for its operation [1–8]. Its adjustment involves two so-called circulation conditions [9]. The first being the maximum power transmission of the demagnetized junction; the second being the introduction of the gyrotropy. A property of any junction circulator is its 1 port G-STUB complex gyrator circuit. Its quality factor is a property of the gyromagnetic resonator, the susceptance slope parameter of the STUB is a property of the geometry of the resonator and the gyrator conductance in this sort of circuit is the dependent variable. All three quantities enter into the design of degree-2 equal ripple circulators.

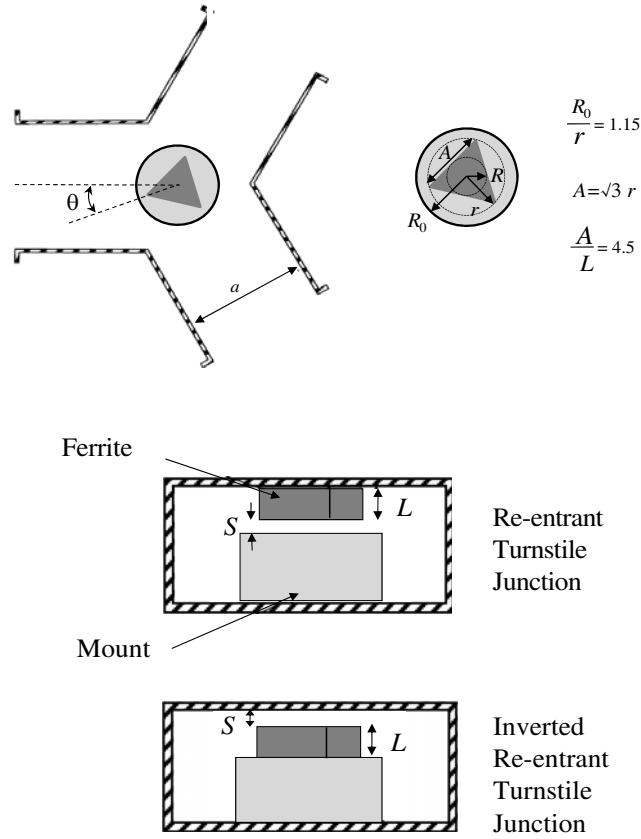
The main activity of this paper is to compare the complex gyrator circuit of a circulator using a prism resonator mounted on a circular platform [5–8] with that mounted on a triangular one. The geometry under consideration has two degrees of freedom, namely, the orientation ( $\theta$ ) of the gyromagnetic prism inside the junction and the aspect ratio,  $A/L$ , of the resonator. The paper deals with the first of these. The general problem dealt with in this paper is that of a quarter-wave long prism resonator short circuited at one flat face and an open wall at the other separated by a gap from the top waveguide wall, mounted on a circular platform, or piston, at the junction of three waveguides. This geometry is known as an inverted re-entrant turnstile junction. The apex of the prism may be oriented at an arbitrary angle,  $\theta$ , with respect to the axis of a typical feed waveguide. The two limiting cases are  $\theta$  equal to 0 deg. and 60 deg. The aspect ratio ( $A/L$ ) is based on some historical commercial practice. This problem, as well as the mode nomenclature, of this class of junction is dealt with in some detail in Ref. [6]. The arrangement under consideration is illustrated in Fig. 1. It differs from that in [9] in that the latter relies on a half-wave long prism open-circuited at each flat face.

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\* Corresponding author: Alicia Casanueva (alicia@dicom.unican.es).

<sup>1</sup> Department of Electrical and Electronic Engineering, Heriot-Watt University, Edinburgh, EH14 4AS, UK. <sup>2</sup> School of Engineering and the Built Environment, Edinburgh Napier University, Merchiston Campus, EH10 5DT, UK. <sup>3</sup> Communication Engineering Department, University of Cantabria, Santander 39005, Spain. <sup>4</sup> Apollo Microwaves, Montreal, QC, H9R 1G6, Canada.



**Figure 1.** Schematic diagram of re-entrant turnstile junction containing an arbitrarily oriented quarter-wave long prism resonator.

The second circulation condition of the circulator is established by replacing the dielectric resonator by a suitably magnetized gyromagnetic one. The degree-1 circulator developed in this way is verified experimentally. The work is undertaken in WR75 at a frequency of 13.25 GHz.

The main conclusion of this work is that the quality factor of the prism resonator mounted on a circular piston is, unlike that mounted on a triangular one, incompatible with the design of quarter-wave coupled degree-2 junction circulators.

## 2. THE TURNSTILE GEOMETRY

The inverted re-entrant turnstile geometry investigated in this paper consists of a single  $E$ -plane quarter-wave long triangular open dielectric waveguide short circuited at one face and has an open flat face separated from the top waveguide wall by a gap. This resonator is mounted on a circular plate, or piston, at the axis of three  $H$ -plane rectangular waveguides. The first circulation adjustment is best described in terms of an eigenvalue problem involving a pair of degenerate counter-rotating travelling wave patterns along the axis of the resonator and one quasi-planar in-phase pattern which does not propagate along the axis. The geometry has, at a fixed frequency, two independent variables, namely, the aspect ratio,  $A/L$  of the resonator, and the orientation angle,  $\theta$ . The impact of  $\theta$  and the cross-section of the piston on the complex gyrator circuit of the junction for a typical value of  $A/L$  are the objectives of this paper.

The first circulation condition defined below is satisfied provided the scattering parameters below are met at the chosen reference terminals [1–4].

$$S_{11} = \frac{1}{3} \quad (1a)$$

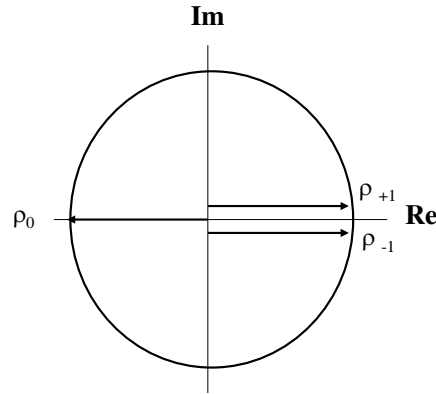
$$S_{21} = \frac{-2}{3} \quad (1b)$$

The corresponding in-phase and degenerate counter-rotating reflection eigenvalues are

$$\rho_0 = -1 \quad (2a)$$

$$\rho_{\pm} = 1 \quad (2b)$$

This solution is illustrated in Fig. 2 [2].



**Figure 2.** Eigenvalue diagram of demagnetized three port junction at the reference terminals.

The former eigenvalue fixes the length of the piston to first order.

The general relationship between the 1-port reflection eigenvalues of the junction and its scattering parameters is given by the bilinear relationship between the two [1, 2].

$$\rho_0 = S_{11} + 2S_{21} \quad (3a)$$

$$\rho_{+1} = \rho_{-1} = \rho_1 = S_{11} - S_{21} \quad (3b)$$

The two dependent parameters of this class of circulator are the side wavenumber,  $(k_0 A)$  and a gap factor,  $(q_{eff})$  of the degenerate counter-rotating reflection eigenvalue of the junction

$$q_{\pm} = \frac{L}{L + S_{\pm}} \quad (4a)$$

The in-phase reflection eigenvalue involves the same field of parameters but with the gap factor  $q_{\pm}$  replaced by  $q_0$

$$q_0 = \frac{L}{L + S_0} \quad (4b)$$

$L$  is the length of the resonator.  $S_0$  and  $S_{\pm}$  are the gaps between the open face of the resonator and the opposite waveguide wall of the in-phase and counter-rotating modes of the junction.

The first condition fixes the gap factor,  $q_{eff}$ , of the junction.

$$q_{\pm} = q_0 = q_{eff} \quad (5)$$

where

$$q_{eff} = \frac{L}{L + S} \quad (6)$$

This adjustment also fixes  $k_0 A$ .

The solution investigated in this paper is restricted to that of a resonator mounted on an oversized circular platform. This situation differs from those in [5–9] in that the cross-section of the platform or piston are there identical.

The adjustment of this class of junction can be either spelled out in terms of  $A$ ,  $L$ , and the gap,  $S$ , between the open face of the resonator and the top wall of the waveguide once the wavenumber  $k_0$  of the junction and waveguide size are specified or in terms of  $k_0$ ,  $A/L$ ,  $k_0 A$ , and  $q_{eff}$ . The calculations involve the former variables but the results of this work are summarized in terms of the latter notation.

### 3. ORIENTATION OF THE PRISM RESONATOR

An efficient means of characterizing any turnstile junction is to use either an FE or mode matching engine [4, 11]. This is done here with  $\theta$  equal to 0 deg, 15 deg, 30 deg 45 deg and 60 deg, in the case of a quarter-wave long prism resonator in an inverted re-entrant turnstile geometry for one value of the aspect ratio  $A/L$  of the prism.

The work undertaken here is in WR75 at a frequency of 13.25 GHz. The cut off frequency,  $f_c$ , of the waveguide completes the specification.

$$\frac{f_0}{f_c} = 1.683$$

The radius of the piston is in the first instance taken as

$$\frac{R_0}{r} = 1.15$$

The aspect ratio is taken on the basis of some prior art as [9]

$$\frac{A}{L} = 4.5$$

The solution, Table 1, is obtained by taking  $R_0/r$ ,  $k_0$ ,  $A/L$ , and the reference plane as the independent variables and  $k_0A$  and  $q_{eff}$  as the dependent ones.

**Table 1.** First circulation condition of inverted re-entrant waveguide circulator using a prism resonator ( $k_0 = 0.277$  rad/mm,  $A/L = 4.5$ ,  $\varepsilon_f = 15$ ,  $R_0/r = 1.15$ ).

$\theta$	$q_{eff}$	$k_0A$	$A$	$L$	$S$
0	0.587	1.680	6.0550	1.3456	0.947
15	0.584	1.680	6.0523	1.3450	0.958
30	0.5825	1.679	6.0505	1.3446	0.964
45	0.583	1.678	6.0450	1.3433	0.961
60	0.59	1.676	6.0400	1.3422	0.933

The ratio of the midband frequency to that of the triangular waveguide cutoff frequency is

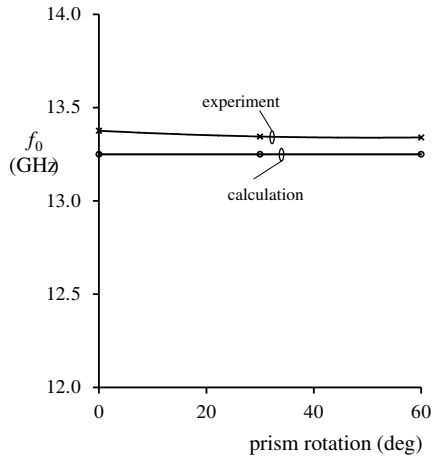
$$\frac{f_0}{f_c} = \frac{k_0}{k_c} = 1.55$$

The cut-off number  $k_c$ , of a prism dielectric waveguide with side length,  $A$ , and perfect magnetic walls is available in [6, 8]. The assumption here is that it coincides with the first pair of  $TM_{1,0,-1}$  degenerate counter-rotating field patterns in the waveguide.

$$k_c = \frac{4\pi}{3A\sqrt{\varepsilon_f}} \quad (7)$$

The experimental and calculated centre frequencies of the junction containing a prism resonator on an oversized circular post as a function of the prism orientation are compared in Fig. 3. The two are also in good accord. A property of the geometry under consideration is that the side dimension of the resonator is, at the design frequency, largely independent of the angle,  $\theta$ . Fig. 4 is a photograph of the hardware.

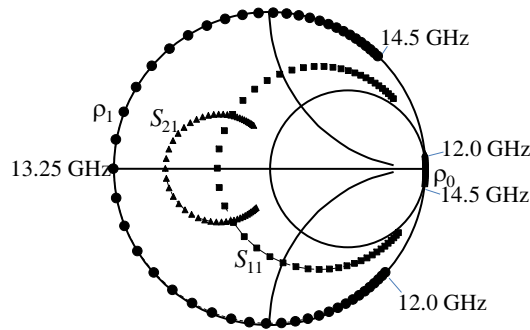
In practice, the junction circulator supports both planar and turnstile dominant and higher order modes. A shortcoming of the eigenvalue calculation is that it does not at first sight identify any particular solution. To do so would be necessary to relationships between  $(k_0A)^2$  and  $(A/L)^2$ . This work is outside the remit of this work. The junction is dealt with here as a turnstile geometry.



**Figure 3.** Calculated and experimental centre frequencies versus prism rotation. ( $A/L = 4.5$ ).



**Figure 4.** Photo of re-entrant turnstile junction using a prism resonator.



**Figure 5.** Simulated Smith Chart response of  $s_0$ ,  $s_1$ ,  $S_{11}$  and  $S_{21}$  over the frequency range 12.0 GHz to 14.5 GHz ( $\theta = 30$  deg).

#### 4. FREQUENCY RESPONSES OF DEMAGNETIZED AND MAGNETIZED PRISM JUNCTIONS

One activity of this work is to deduce the susceptance slope parameter of the 3-port junction circulator. This quantity may be extracted from either the frequency response of the reciprocal junction or from that of the magnetized one. This section summarizes the passband responses of each arrangement. This is done as a preamble to some calculations on the susceptance slope parameter of the device.

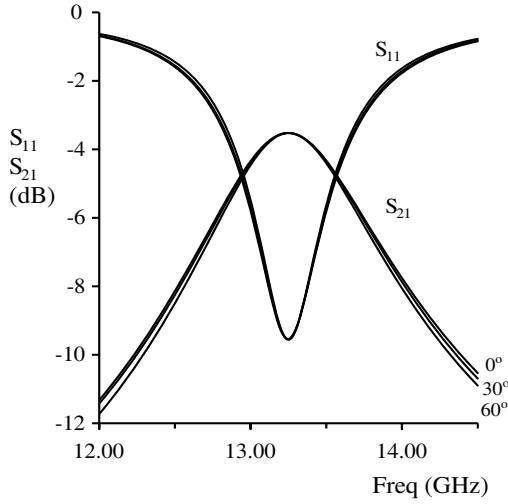
Figure 5 is a Smith Chart of the scattering parameters and in-phase and counter-rotating reflection eigenvalues of the solution.

A scrutiny of this Smith Chart indicates that the frequency response of the in-phase admittance eigenvalue may be neglected compared to those of the degenerate ones. This feature is in keeping with the usual relationship associated with the classic turnstile circulator. The return losses of the demagnetized and magnetized junctions are illustrated separately in Figures 6 and 7.

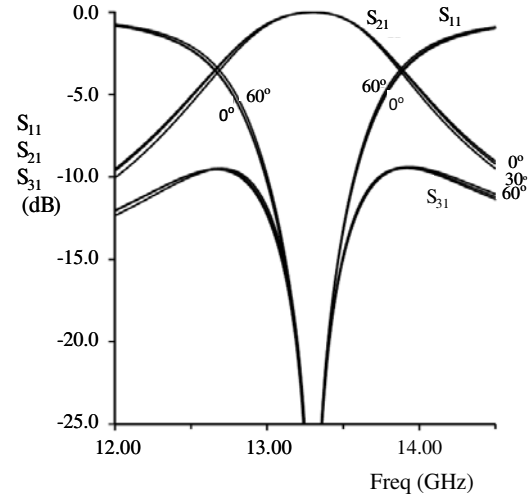
Figure 6 shows the frequency response of the demagnetized junction for  $\theta = 0, 30$  and  $60$  deg about 13.25 GHz.

The frequency response of the magnetized device is indicated in Fig. 7. These results, in keeping with the Table 1, suggest that the angle,  $\theta$ , has no, or very little impact on the performance of both the demagnetized and magnetized junctions.

The eigenvalue diagram at the midband frequency of the degree-1 circulator response in Fig. 7 is shown for completeness in Fig. 8 [2].



**Figure 6.** Simulated frequency response of 3-port reciprocal junction using prism resonator ( $\theta = 0, 30, 60$  deg).



**Figure 7.** Simulated frequency response of 3-port junction circulator using prism resonator ( $\theta = 0, 30, 60$  deg).

## 5. SUSCEPTANCE SLOPE PARAMETER

A fundamental quantity that is met in the description of the 1-port STUB-G complex gyrator circuit of the junction circulator, besides its quality factor, is the susceptance slope parameter ( $b'$ ) of the STUB. It enters into the design of any degree-2 circulator. Its real and imaginary parts may be expressed in terms of linear combinations of the split counter-rotating 1-port admittance eigenvalues provided the frequency variation of the in-phase eigenvalue may be neglected. The definition of the 1-port complex gyrator admittance of the junction is [14]

$$y_{in} = \left( \frac{y_+ + y_-}{2} \right) - j\sqrt{3} \left( \frac{y_+ - y_-}{2} \right) \quad (8)$$

$y_{\pm}$  are here pure imaginary numbers

$$y_{\pm} = jb_{\pm} \quad (9)$$

The two quantities that describe this type of circuit are its susceptance slope parameter ( $b'$ ) and its loaded quality factor ( $Q_L$ )

$$b' = \left( \frac{\omega_0}{2} \right) \frac{\partial b_{in}}{\partial \omega} \bigg|_{\omega=\omega_0} \quad (10)$$

$$Q_L = \frac{b'}{g} \quad (11)$$

$g$  is here the real part of  $y_{in}$ .

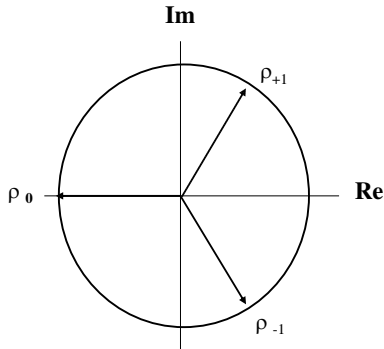
The susceptance slope parameter is evaluated between the 9.5 dB return loss points at port 1 of the junction with ports 2 and 3 terminated in matched loads. The result is

$$b' = \frac{\omega_0}{2} \left( \frac{b_- - b_+}{\omega_+ - \omega_-} \right) \quad (12)$$

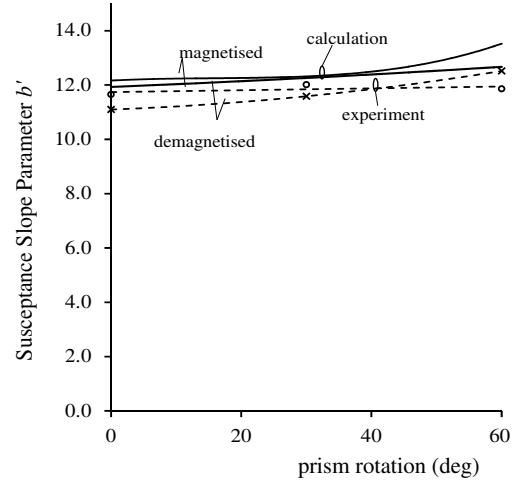
$\omega_{\pm}$  are the split frequencies of the gyromagnetic resonator at the 9.5 dB points of the return loss at port 1.

$$y_{in} = y_+ \quad \text{at } \omega = \omega_- \quad (13a)$$

$$y_{in} = y_- \quad \text{at } \omega = \omega_+ \quad (13b)$$



**Figure 8.** Eigenvalue diagram at the midband frequency of the ideal degree-1 circulator.



**Figure 9.** Calculated and experimental Susceptance slope parameter of reentrant turnstile junction prism resonators versus prism rotation angle.

The required result is now established by eliminating the difference between the split susceptances in the above equation in terms of the gyrator conductance  $g$  by making use of Eq. (11). This gives

$$Q_L = \left[ \sqrt{3} \left( \frac{\omega_+ - \omega_-}{\omega_0} \right) \right]^{-1}, \quad R.L. = 9.5 \text{ dB} \quad (14)$$

and

$$b' = Q_L g, \quad g = 1, \quad R.L. = 9.5 \text{ dB} \quad (15)$$

The gyrator conductance  $g$  is unity in a degree one circulator.

Combining the previous two equations gives an experimental method for the determination of the susceptance slope parameter.

$$b' = \left[ \sqrt{3} \left( \frac{\omega_+ - \omega_-}{\omega_0} \right) \right]^{-1}, \quad g = 1, \quad R.L. = 9.5 \text{ dB} \quad (16)$$

The susceptance slope parameter ( $b'$ ) of the demagnetized junction may also be extracted from the slope of the degenerate counter-rotating 1-port admittance eigenvalues ( $y_1$ ) in the Smith chart in Fig. 5. This may be done provided the dispersion of the in-phase one ( $y_0$ ) can be, as in this case neglected and provided.

$$\frac{y_+ + y_-}{2} = y_1 \quad (17)$$

The susceptance slope parameter is the same as in Eq. (10) but with  $b$  replaced by  $y$ . The frequency interval here is, arbitrarily, taken between the 6 dB return loss points at port-1 with ports 2 and 3 terminated in matched loads.

The susceptance slope parameter, based on the calculations of the split frequencies of the magnetized junction and on the return loss of the demagnetized one, are indicated in Fig. 9. Measured results on the three fabricated prototypes on each experimental procedure are separately superimposed on this illustration. The agreement between the two procedures is of note. The ferrite material employed in obtaining this result is an Yttrium Iron Garnet one with a saturation magnetization  $\mu_0 M_0 = 0.1780 \text{ T}$ . The relative dielectric constant of the magnetic insulator is 15.3.

This result may be compared to some data elsewhere on a geometry based on a half wave long prism gyromagnetic resonator, open-circuited at both flat faces, supported by dielectric spacers. It too shows a second order dependence of the susceptance slope parameter of the junction on the two limiting orientations of the resonator. The values there in the cases of the apex and side coupled geometries are 6.7 and 7.5 respectively [9]. The difference between the susceptance slope parameters of a half wave

and quarter wave resonator is a factor of two [11]. The resonator there is mounted symmetrically on triangular platforms or pistons. The susceptance slope parameter of this class of circulator has also been investigated in WR90 waveguide [11]. It displays a somewhat larger deviation between the two values of the quantity in question for each of the two possible orientations of the prism. It differs however in that the resonator is mounted on a triangular platform rather than on a circular one. Whereas it has the same aspect ratio as used here its side wavenumber and its gap factor are different. The absolute values obtained there are of the same order as those obtained here.

## 6. QUALITY FACTOR

Figure 10 depicts some calculations on the quality factor of various resonator structures. It suggests that the usual structure is the most beneficial for the design of conventionally coupled quarter wave circulators [10, 12, 13, 15]. This result represents a significant insight into the operation of this class of circulator. The magnetization used in this class of devices is normally expressed in terms of a normalized quantity.

$$p = \frac{\gamma M_0 \mu_0}{\omega_0}$$

$M_0$  is the saturation magnetization of the magnetic insulator (A/m),  $\mu_0$  is the free space permeability  $4\pi \times 10^{-7}$  (H/m),  $\gamma$  is the gyromagnetic ratio  $2.21 \times 10^5$  (rad/s/A/m), and  $\omega_0$  is the radian frequency (rad/s).

The value of the magnetization ( $\mu_0 M_0$ ) used here is 0.3200 T. The value of  $p$  is usually bracketed between 0 and 0.85 the value used here is 0.674. There is in practice a trade-off between insertion loss and  $p$ .

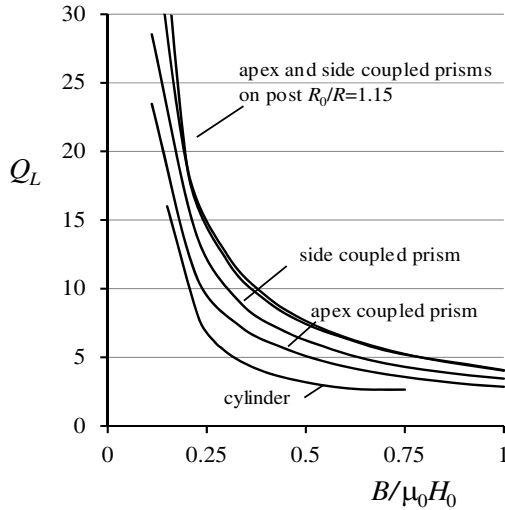
The split cut-off numbers of prism and cylindrical resonators with closed side wall are [7, 16]

$$\left( \frac{\omega_+ - \omega_-}{\omega_0} \right) = \left( \frac{\sqrt{3}}{\pi} \right) \left( \frac{\kappa}{\mu} \right)$$

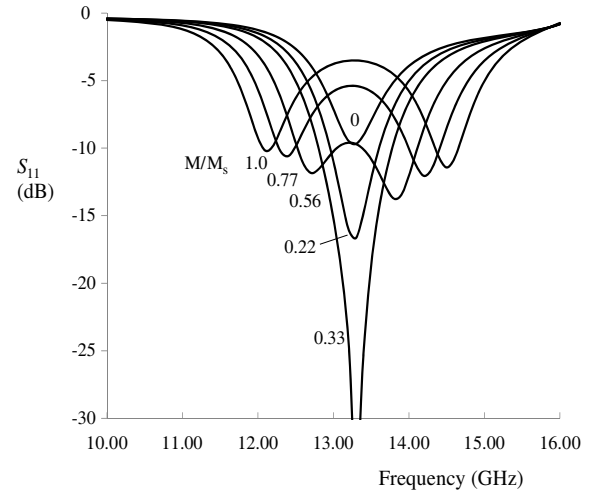
and

$$\left( \frac{\omega_+ - \omega_-}{\omega_0} \right) = \left( \frac{(kR)^2 - 1}{2kR} \right) \left( \frac{\kappa}{\mu} \right)$$

A scrutiny of these two relationships indicates that the opening between the split frequencies is less for the prism than it is for the cylinder, in keeping with the data here.



**Figure 10.** Simulation of the effect of piston shape and orientation on quality factor,  $Q_L$ .



**Figure 11.** Simulated split frequencies of an apex coupled prism resonator on a prism mount.



## 7. THE GYRATOR CONDUCTANCE

There also exists a classic relationship between the gyrator conductance, the susceptance slope parameter and the split frequencies of the gyromagnetic resonator. It is defined without ado by making use of the equation

$$g = \sqrt{3}b' \left( \frac{\omega_+ - \omega_-}{\omega_0} \right) \quad (18)$$

$g$  is here the dependent variable. A typical graphical depiction of the split frequencies is shown in Fig. 11.

This relationship is valid provided again that the frequency variation of the in-phase eigen-network may be neglected compared to that of the counter-rotating ones.

The susceptance slope is here, as already noted, a constant of the geometry and the operating mode of the resonator. The quality factor is a property of the gyrotropy of the gyromagnetic resonator. A typical value of  $Q_L$  is 2.5 for the material employed in this work. The corresponding gyrator conductance of the geometry under consideration is 4.

## 8. CONCLUSION

A gyromagnetic resonator met in the design of the  $H$ -plane turnstile waveguide circulator is an arbitrarily oriented prism geometry. Its 1-port complex gyrator circuit is the topic of this paper. The situation described in this paper differs from that previously investigated in that here the prism sits on a circular rather than on an equilateral triangular platform. The agreements between theory and measurements on both demagnetized and magnetized junctions are excellent and also compatible with historic data. The quality factor of the arrangement under consideration is significantly different from that of a similar resonator mounted on a triangular piston.

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